**RESEARCH NOTE**

**Comparative Politics vs. Legal Perspectives on Measuring Malapportionment:**

**With Applications to the U.S. House, the U.S. Senate and the U.S. Electoral College,**

**1790-2010**

**ABSTRACT**

**The comparative politics approach to measuring malapportionment has been in terms of an adaptation of standard measures of seats-votes discrepancy, such as the *Loosemore-Hanby Index of Distortion* (1971), used by Samuels and Snyder (2001) and Sauger and Grofman (2016) among others. This approach examines malapportionment levels as whole, taking into account the differences in each of the districts between actual population and ideal population. In contrast, U.S. courts, and courts in many other countries, including Canada, France and Germany, measure compliance with a one person, one vote standard using what is called the *total population deviation* (TPD), which looks only at the two extreme poles of malapportionment, i.e., the difference between actual population and ideal population in the smallest district and in the largest district. We demonstrate that which approach we take can dramatically affect the conclusions we reach, illustrating this point with data from the U.S. House, Senate, and Electoral College over the period 1790-2010. In particular, using TPD, the Electoral College (EC) is substantially malapportioned; under the Loosemore-Hanby Index and the closely related Gallagher Index (Gallagher 1991), it is not. In contrast, the U.S. Senate is malapportioned no matter how we measure malapportionment. We show that we can explain these difference in measurement results once we realize that the states with the greatest malapportionment do not make up a large share of the EC vote. While as a matter of mathematical necessity, malapportionment in the EC must be larger than malapportionment in the U.S. House, it is not that much larger, and the EC is far less malapportioned than the U.S. Senate.**

**Keywords: Malapportionment, Electoral College, *Total Population Deviation*, *Gallagher Index*, *Gini Index*, Elections, Representation, One person, one vote, Voting**

# I. Introduction

**Malapportionment, in and of itself, may or may not have direct pernicious consequences for the treatment of particular political parties or cognizable groups of voters with distinct interests. For example, Singapore has high levels of malapportionment, but that malapportionment does not appear to have effects that favor the ruling party, the PAP (Tan and Grofman, 2016). In contrast, malapportionment in Japan has historically favored rural areas by over-representing rural voters, and thus been a boon to the LDP whose greatest strength came from rural voters (BG FILL IN REF). In the U.S., although malapportionment bias is often regarded as inherently normatively undesirable,[[1]](#footnote-1) it was the perceived effects on government policies stemming from underrepresentation of city dwellers that motivated much of the sentiment that agitated pre-*Baker v. Carr* (1962) reformers (see e.g. Baker, BG FILL IN).[[2]](#footnote-2) But, regardless of why we regard malapportionment as problematic, we must answer the question: “How do we measure malapportionment?” [[3]](#footnote-3)**

**Students of American politics writing before and just after *Baker v. Carr* (1962) considered a number of different ways to measure malapportionment. These included the *total population deviation*, which looks at the difference in population between the most underpopulated and the most overpopulated district; the *population deviation ratio*, which is the ratio of the population in the largest district to that in the smallest, average absolute level of malapportionment; and the minimum population share needed to control a majority of seats in the legislature.**

**For simplicity of exposition, we present below definitions of these four measures for the case of single seat constituencies. Let = ideal population, i.e., the total population divided by the number of seats in the legislature, pi = population in the ith constituency, pL = population in the constituency with the largest population, pS = population in the constituency with the smallest population, n = number of constituencies.**

**Total population deviation (TPD) = (pL - pS)/**

**The TPD measure is sometimes written as ( –pS)/ ) +(pL- )/ .**

**Total population ratio (TRD) = pS / pL**

**Average malapportionment = Σ │pi -│/n**

**Finally, to find the *minimum population share needed to control a majority of the seats* in the legislature, for the case of single seat constituencies, we order the districts from smallest to largest in population, and continue until we find the median district. We find the population of the districts up to and including the median district and then divide by the total population to obtain the proportion we seek**.**[[4]](#footnote-4)**

**At the beginning of the Reapportionment Revolution, considering evidence presented on these measures and others at the trial level, U.S. courts, and ultimately the U.S. Supreme Court, seized upon the first of these measures as the appropriate legal standard. The TPD measure is conceptually very simple, and like the other three measures it can be used to specify a *de minimis* threshold that can serve as a bright line test. Over the course of two decades, the U.S. Supreme Court set up a hierarchy, with the tightest standard for the U.S. Congress, as close to zero TPD as practicable; [[5]](#footnote-5) the next highest standard for state legislative redistricting, a TPD of 10%;[[6]](#footnote-6) and the lowest standard for local districting, where TPD values as high as 16.4% have been permitted. [[7]](#footnote-7) Virtually every other democracy which imposes some form of one person, one vote test on its parliamentary constituencies has also adopted a TPD bases measure, though with widely differing thresholds, with most far higher than the ones adopted in the U.S. e.g., 30% in Germany and 50% in Canada.[[8]](#footnote-8)**

**In contrast, when students of comparative politics study redistricting they have used a different measure of malapportionment, one adapted from the electoral systems literature on measuring the discrepancy between party vote share and party seat share, the *Loosemore-Hanby Index of Distortion* (1971). This measure, along with the closely related *Gallagher Index* (1991), are the two most common metrics**[[9]](#footnote-9)  **used for measuring seats-votes disproportionality.** *Loosemore-Hanby* measures the summed absolute differences between seats and votes, while *Gallagher’s index*, often referred to as a Least Squares measure, weights each observation by the size of the deviation, i.e., it squares the deviations. Squaring the deviations puts more weight on larger deviations, while discounting smaller ones. The analogues to these two disproportionality indices in the malapportionment context are shown below.

Samuels and Snyder (2001: 654) discuss the reasons they chose to use the *Loosemore-Hanby Index* to measure malapportionment in the seventy-eight countries whose malapportionment they examine.

Ratios of largest-to-smallest districts might seem an obvious means for

assessing malapportionment. However, such ratios actually prove poor

indicators of malapportionment. First, district size on the basis of population

tells us little about the degree to which districts are underrepresented or

overrepresented: we also need to know how many seats are allocated to each

district. Furthermore, even if we know how many seats are held by the largest

and smallest districts and can therefore calculate ratios of 'worst represented'

to 'best represented' districts, such ratios tell us little about overall degrees of

rnalapportionment. For example, even if this ratio is 50:1(e.g., a single-member

district system in which the largest district has a population fifty times greater

than the smallest district), all other districts may have nearly-equivalent

populations, and, hence, the largest and smallest districts could be extreme

outliers in a system with a low degree of average malapportionment. Although

it may be tempting to interpret wide gaps between the best and worst represented

districts as signs of high overall levels of inequality in electoral systems, a better

measure is required. …[T]he Loosemore-Hanby index of electoral

disproportionality (D) provides such a measure (*internal footnotes omitted*) .

**But the measure of malapportionment used by courts and legislatures in virtually all the major democracies worldwide, the TPD, is exactly the sort of measure of the gap between the most overpopulated and the most underpopulated districts that Samuels and Stewart (2001: 654) warn against. So, a natural question to ask is: “How much does the choice of malapportionment measures chosen affect the conclusions we reach about level of malapportionment?”**

**We will address this question with U.S. data over the period 1790 to 2010 on the U.S. House of Representatives, the U.S. Senate and the U.S. Electoral College. In order to calculate malapportionment in the Electoral College we treat each state as a unit and look at the weighted differences or ratios between actual population and assigned EC seat share ,JONATHAN IS THIS CORRECT, OR DO WE NEED TO EXPAND ON THIS POINT?**

The structure of Electoral College seat allocation makes it mathematically guaranteed that EC malapportionment is in between that for the House and that for the Senate. **Under the six metrics identified above, we show empirically that, exactly as we would expect, malapportionment is greatest in the Senate, next largest in the Electoral College, and smallest in the House of Representatives.[[10]](#footnote-10) We show that the differences in level of malapportionment across the three types of elections has varied over the historical time period we review regardless of which measure we use but that there is greater variability under some measures than under others. However, regardless of metric,** apportionment equality in the EC, judged at the state level, looks far more like apportionment equality for the U.S. House of Representatives than it looks like apportionment inequality in the U.S. Senate**.**

In this context it is useful to remember that, while, today, House districts are almost identical in population to one another within any given state, even so, the combination of apportionment rounding rules (the so-called *integer allocation problem:* Balinski and Young, 1982) and the rule that no state can be denied a seat in the House of Representatives regardless of its population, introduces malapportionment into the U.S. House when malapportionment is calculated, nationally and not state by state (Ladewig and Jasinski, 2008). [[11]](#footnote-11) **Moreover, and most importantly for purposes of the present research note, which measure(s) of malapportionment we use will substantially affect our views of the seriousness of the malapportionment problem in these three types of elections, although we find that the Senate will almost certainly be judged as malapportioned regardless of what measure we use.**

Figure 1 (a-f) show the comparisons across the three types of elections for each of our six metrics.

<Figure 1 about here>>

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Since the first two of our measures focus on the same two states, the two EC disproportionality measures defined above are quite highly correlated, with a correlation of 0.90. However, when we compare Figures 1a and 1b, EC *Total Population Deviation* appears toshow a more muted effect, because we are directly taking ideal population into account. But both Figures 1a and 1b reinforce our claim that EC malapportionment is closer to low levels of House malapportionment than it is to the high levels of Senate malapportionment. The next four draw on information for all the districts, **and further reinforce the point that EC malapportionment is far closer to that of the House than it is to that of the Senate.**

**Figure 1a: Total Population Deviation for Electoral College Votes, 1790-2010, with Comparisons to the U.S. House and the U.S. Senate**

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**Figure 1b: Ratio of the Largest and the Smallest State by EC Seat Share versus State Population Share: 1790-2010, with Comparisons to the U.S. House and the U.S. Senate**

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**Figure 1c: Average Malapportionment of EC Seat Share versus State Population Share: 1790-2010, with Comparisons to the U.S. House and the U.S. Senate**

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**Figure 1d: Minimum Proportion of the Population Need to Secure Electoral College Majority, with Comparisons to the U.S. House and the U.S. Senate**

**JONATHAN THIS NEEDS TO BE IN THE EXACT SAME FORMAT AS THE PREVIOUS THREE TABLES, I.E., WITH PARALLEL DATA FOR THE HOUSE AND THE SENATE. ALSO IN THE NEXT VERSION PLEASE ONLY USE THE DARK LINE AND PROVIDE A FOOTNOTE TO EXPLAIN THE COMPLEXITIES.**

**../Dropbox/EC%20Grofman%20Cervas/minEC.pdf**

**Figure 1e: Loosemore-Hanby Index for the Electoral College, with Comparisons to the U.S. House and the U.S. Senate**

**JONATHAN THIS NEEDS TO BE IN THE EXACT SAME FORMAT AS THE PREVIOUS THREE TABLES, I.E., WITH PARALLEL DATA FOR THE HOUSE AND THE SENATE.**

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**Figure 1f: Gallagher Index for the Electoral College, with Comparisons to the U.S. House and the U.S. Senate**

**JONATHAN THIS NEEDS TO BE IN THE SAME FORMAT AS THE PREVIOUS THREE TABLES, I.E., WITH PARALLEL DATA FOR THE HOUSE AND THE SENATE.**

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**When we compare across all six of our metrics we find that measures that focus on the best and worst cases (the first two measures) show considerably higher malapportionment effects than measures or the average population deviation from ideal, or the minimum proportion needed to control a majority of seats, or measures adapted from seats-votes disproportionality measures, such as the *Loosemore-Hanby Index*, the absolute deviations from ideal summed over all districts, or the *Gallagher Index*. Moreover, all but JONATHAN FILL IN WHEN YOU HAVE ALL THE CHARTS COMPLETED. of the measures suggest that malapportionment has been falling over the course of U.S. history in the House, the Senate and the Electoral College, and has been remarkably flat the past several decades. JONATHAN REVISIT THIS SENTENCE WHEN YOU HAVE ALL THE CHARTS COMPLETED.**

**Indeed, if we compare the current values we get for those measures to their equivalents in the seats vote disproportionality context, the U.S. House and even the U.S. Electoral College exhibit low levels of disproportionality. In fact, the numbers shown in Figures 1e and 1f for the Gallagher index, while not as small as the party-based disproportionalities reported for the most highly proportional electoral rules in use world-wide, those of Netherlands[[12]](#footnote-12) and Israel[[13]](#footnote-13), are comparable to the partisan disproportionalities in other western European democracies.** For example, tabulating data from Doring and Manow (2015: Table 3.p. 159) shows that proportional countries have an average *Gallagher* least squares disproportionality of 2.4, while majoritarian counties average 9.6. The *Gallagher* least squares disproportionality in the Electoral College in 2016 was just 2.06.

# II. Discussion

**While we find Samuels and Snyder’s (2001: 654) preferences for the *Loosemore-Hanby Index* over measures focusing on only the districts with highest and lowest populations relative to their seat allocations to be based on sensible grounds, we are less willing to dismiss the wisdom of courts in dozens of countries who have adopted the Total Population Deviation as their measure. To return to the Samuels-Snyder example of a constituency that is 1/50th the population of the largest district, even if all the other districts are very close in population, and the legislature is large enough that the existence of such a dramatically underpopulated district does not affect that much the average population deviation in an environment where almost all of the other districts are of very much the same population size, having such a discrepancy left untouched seems very problematic. Moreover, if you force the largest and the smallest district to be not that far from ideal population size, *ipso facto*, you also reduce average population deviation. Nonetheless, we have demonstrated that the choice of malapportionment index makes a tremendous difference in how we evaluate the level of malapportionment in the U.S. Electoral College, even though it matters little for the U.S. Senate. JONATHAN THINK ABOUT WORDING OF LAST SENTENCE[[14]](#footnote-14)**

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**JONATHAN MANY OF THESE ARE NO LONGER IN THE RESEARCH NOTE. CAN YOU DOUBLECHECK refs to delete unused ones and make sure that anything cited is in the refs?**

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1. For example, Dahl (1971) regards the one person, one vote principle as a necessary component of democratic governance, and Taagepera and Shugart (1989) consider malapportionment as a pathology. [↑](#footnote-ref-1)
2. **Samuels (2001: 653) reviewing a number of single country studies, concludes “malapportionment can have an important impact on executive-legislative relations, intra-legislative bargaining and the overall performance of democratic systems.”** [↑](#footnote-ref-2)
3. **Addressing the partisan consequences of malapportionment in the Senate or in the Electoral College, or its past consequences for the U.S. House of Representatives is outside the scope of this research note.** [↑](#footnote-ref-3)
4. **Sometimes this fraction is divided by two in order to indicate that only a majority of the votes in each constituency are needed to control the outcome in that constituency, but we prefer not to introduce that further complication.** [↑](#footnote-ref-4)
5. **BG add ref The justification of the higher standard for congressional districting was that it was rooted in Article I defining the role of the House as the repository of the popular principle of representation, while justification for state and local malapportionment standards were found in the Equal Protection clause of the 14th Amendment.** [↑](#footnote-ref-5)
6. **BG add ref . *Connor v. Finch* Until *Larios v. Cox*, BG add ref a TPD of 10% was taken to be a safe harbor for jurisdictions, with no need to justify deviations as long as the TPD fell below the 10% threshold. In that case the Supreme Court held that a systematic pattern of underpopulation and overpopulation motivated solely by partisan intent was not constitutional even if the TPD fell (marginally) below 10%.** [↑](#footnote-ref-6)
7. **BG add ref A TPD above 10% would be permitted only if the jurisdiction could offer legitimate (and compelling) reasons for the deviation** [↑](#footnote-ref-7)
8. **See Handley and Grofman (2008) for a review of legal malapportionment thresholds in many countries. However, the reader must be careful in interpreting reported thresholds. For example, the threshold in Germany is stated as no more than 15% upwards or downwards from the average, and those who write about Germany may thus correctly characterize it as a 15% tolerance limit but, in our terms, this is a 30% TPD value. Also there are many complexities in defining malapportionment when we move from simple single seat systems to countries with multi-seat districts and/or a mix of single and multiple seat districts, and/or a tiered system with proportional allocations or compensatory seats in the upper tier. Samuels and Snyder (2001) offer the best discussion of how to deal with such complexities.**  [↑](#footnote-ref-8)
9. There are, however, many other measures that have been proposed. It is well recognized that no single measure of disproportionality can capture every feature of interest, and each measure has some desirable properties and some flaws. (Cox and Shugart 1991, Monroe 1994; Taagepera and Grofman, 2003). [↑](#footnote-ref-9)
10. The same is true for a seventh metric, the Gini Index. For space reasons we do not present the Gini Index results here. They are available upon request from the authors. [↑](#footnote-ref-10)
11. Malapportionment across states can also occur for the U.S. House of Representatives when Congress fails to fulfill its decennial duty to reapportion the House in accord with new population data. After the 1920 census Congress failed to reapportion the House in accord with new census data (<https://www.census.gov/history/www/reference/apportionment/apportionment_legislation_1890_-_present.html>). The failure to reapportion after the 1920 census came because of controversy over the need to transfer seats from more rural states whose population was falling, in relative terms, compared to heavily urban states with growing populations. Reapportionment was resumed in 1930 and a rule was set in place that provided for automatic reapportionment after in each census in accord with a specified apportionment formula that provided population based reapportionments. While that formula was changed for the 1940 census, and a still different formula had been used early in the nation’s history <https://www.census.gov/population/apportionment/about/history.html>, the differences in allocation across apportionment formulae, tend to be minor (see Balinski and Young, 1982). Within states, malapportionment of congressional seats or legislative seats can occur when the redistricting that does take place violates principles of one person, one vote. Prior to *Baker v. Carr* (1962) and the subsequent one person, one vote cases, especially for state legislatures, this could be a major problem. Some state constitutions, such as that in Georgia, which used counties as its redistricting units, had state constitutional provisions that required redistricting that was not entirely (or even mostly) based on population. But even absent constitutional provisions that forced malapportionment, some states simply refused to reapportionment. For example, Tennessee, whose constitution required decennial redistricting, but which had not redistricted its legislature since 1901, had its grossly malapportioned legislative districts challenged in *Baker v. Carr* (1962), and the state was compelled to redistrict. Moreover, pre-*Baker v. Carr*, while most state legislatures (especially upper chambers) were more malapportioned than the U.S. House districts in the state, there were also states whose House seats were also severely malapportioned. For example, in 1962 Georgia’s largest House seat had more than twice the population of its smallest House district (Bullock, 2010: 141). [↑](#footnote-ref-11)
12. 2017 Dutch Election: Loosemore-Hanby – 0.013, Gallagher – 0.006. Data source: https://www.kiesraad.nl/ [↑](#footnote-ref-12)
13. 2015 Knesset Election: Loosemore-Hanby – 0.015, Gallagher – 0.007. Data source: https://www.knesset.gov.il [↑](#footnote-ref-13)
14. **Of course, it seems also true that one of the key claimed defects of the Electoral College, the occasional divergence between its outcomes and those of the popular vote, cannot be attributed to malapportionment, but rather to the distribution of partisan electoral support across states in situations where the popular vote is relatively closely divided (IDENTIFYING REFERENCE REMOVED) . JONATHAN THINK ABOUT WORDING**  [↑](#footnote-ref-14)